

Entanglement-fidelity relations for inaccurate ancilla-driven quantum computation

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It was shown in [T. Morimae, Phys. Rev. A **81**, 060307(R) (2010)] that the gate fidelity of an inaccurate one-way quantum computation is upper bounded by a decreasing function of the amount of entanglement in the register. This means that a strong entanglement causes the low gate fidelity in the one-way quantum computation with inaccurate measurements. In this paper, we derive similar entanglement-fidelity relations for the inaccurate ancilla-driven quantum computation. These relations again imply that a strong entanglement in the register causes the low gate fidelity in the ancilla-driven quantum computation if the measurements on the ancilla are inaccurate.

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I. INTRODUCTION

In the circuit model [1] of quantum computation, the quantum register which stores quantum information consists of many qubits, and a quantum gate operation is performed by directly accessing one or two qubits in the quantum register. The most challenging task for an experimentalist who adopts the circuit model is therefore the coherent establishment of entanglement among register qubits in parallel with the execution of a quantum algorithm. From the theoretical point of view, the role of entanglement played in the circuit model of quantum computation has been the most fundamental subject of study [2, 3].

In the one-way model [4] of quantum computation, on the other hand, a highly entangled state which is called the “cluster state” (or the “graph state”) is prepared in advance and the whole quantum computation is performed by adaptive measurements of each qubit. The preparation of the resource (i.e., entanglement) is thus clearly separated from the consumption of the resource. This great advantage of the one-way model has lead to many experimental implementations [5–11] and theoretical investigations about the roles of entanglement in the one-way quantum computation [12–16].

Recently, a mixture of those two models, which is called the ancilla-driven quantum computation, was proposed [17, 18]. In this model, the quantum register is a set of many qubits like the circuit model. However, a quantum gate operation is, like the one-way model, performed by using entanglement and measurements: one or two register qubits are coupled to a single mobile ancilla, and the ancilla is measured after establishing the interaction between the ancilla and register qubit(s). The backaction of this measurement provides the desired gate operation, such as a single qubit rotation or an entangling two-qubit operation, on register qubit(s) (see Fig. 1). The main feature of this model is that the uni-

versal quantum computation is performed with only a single type of interaction (CZ or CZ+SWAP) between the ancilla and register qubit(s). It is advantageous to some experimental setups where the implementation of various types of interactions at the same time is very difficult (such as the solid-based quantum computation) or where the flying ancilla mediates interactions between static qubits (such as the chip-based quantum computation [20] or the hybrid quantum computation of matter and optical elements [19]).

In this paper, we study how entanglement among register qubits affects the gate fidelity in the ancilla-driven quantum computation if the measurement is inaccurate. For this purpose, we generalize the result of Ref. [16] to an inaccurate ancilla-driven quantum computation. In Ref. [16], the relation

$$F \leq 1 - S \sin^2 \frac{\epsilon}{2} \quad (1)$$

between entanglement S , the gate fidelity F , and the inaccuracy ϵ of the measurement was derived for the inaccurate one-way model (for details, see Sec. III). The meaning of this inequality is that if the entanglement is strong, the inaccurate measurements make the gate fidelity low.

The results of this paper are: (I) For the ancilla-driven single-qubit rotation, we obtain the same entanglement-fidelity relation as given in Eq. (1). (II) For the ancilla-driven two-qubit entangling gate with the CZ interaction, we again obtain the same entanglement-fidelity relation as given in Eq. (1). (III) For the ancilla-driven two-qubit entangling gate with the CZ+SWAP interaction, we obtain the entanglement-fidelity relation Eq. (8) which is slightly different from Eq. (1). However Eq. (8) also implies that if the entanglement is strong, the inaccurate measurements make the gate fidelity low.

This paper is organized as follows: We will briefly review the ancilla-driven quantum computation [17, 18] in Sec. II for the convenience of the reader. In Sec. III, we will review and extend the result of Ref. [16] about the entanglement-fidelity relation for the inaccurate one-way model. We will study the entanglement-fidelity relation

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for the inaccurate ancilla-driven single-qubit rotation in Sec. IV A. We will also study the entanglement-fidelity relation for the inaccurate ancilla-driven two-qubit entangling gates with the CZ interaction in Sec. IV B and that with the CZ+SWAP interaction in Sec. IV C, respectively.

Throughout this paper, \hat{X}_i , \hat{Y}_i , and \hat{Z}_i are Pauli x , y , and z operators on i th qubit, respectively. $\hat{1}_i$ is the identity operator on i th qubit. Eigenvectors of them are $\hat{X}|\pm\rangle = \pm|\pm\rangle$ and $\hat{Z}|z\rangle = (-1)^z|z\rangle$ $z \in \{0,1\}$, respectively. \hat{H}_i is the Hadamard operator acting on i th qubit. The Hadamard operator works as $\hat{H}|0\rangle = |+\rangle$ and $\hat{H}|1\rangle = |-\rangle$.

II. ANCILLA-DRIVEN QUANTUM COMPUTATION

Let us briefly review the ancilla-driven quantum computation [17, 18]. As in the case of the circuit model [1], the quantum register is a set of N qubits. Unlike the circuit model, however, a quantum operation on one or two qubits in the register is indirectly driven by a single mobile ancilla which can couple to one or two qubits through a fixed interaction (see Fig. 1). An advantage of this model is that a single type of interaction is sufficient for the universal quantum computation.

For example, it was shown [17, 18] that the interaction

$$\hat{E} \equiv \hat{H}_A \hat{H}_R \hat{C}Z_{A,R}$$

is sufficient for the ancilla-driven universal quantum computation, where \hat{H}_A is the Hadamard operation on the ancilla, \hat{H}_R is the Hadamard operation on a register qubit, and $\hat{C}Z_{A,R}$ is the Controlled-Z (CZ) gate

$$\hat{C}Z_{A,R} \equiv |0\rangle\langle 0|_A \otimes \hat{1}_R + |1\rangle\langle 1|_A \otimes \hat{Z}_R$$

between the ancilla and the register qubit. Indeed, the single qubit rotation by $u \in \mathbb{R}$ about z -axis (plus the Hadamard correction)

$$\hat{J}(u) \equiv \hat{H} e^{i\frac{u}{2}\hat{Z}}$$

is implemented as is shown in Fig. 2 (a). The Hadamard correction is canceled by just implementing

$$\hat{J}(0)\hat{J}(u) = e^{i\frac{u}{2}\hat{Z}}.$$

The rotation by u about x -axis is implemented as

$$\hat{J}(u)\hat{J}(0) = e^{i\frac{u}{2}\hat{X}}.$$

Therefore, according to the Euler decomposition, any single-qubit rotation is possible by using $\hat{J}(u)$. The CZ gate between two qubits is also implemented by using the same interaction E as is shown in Fig. 2 (b). Any single-qubit rotation plus the CZ gate are sufficient for the universal quantum computation.

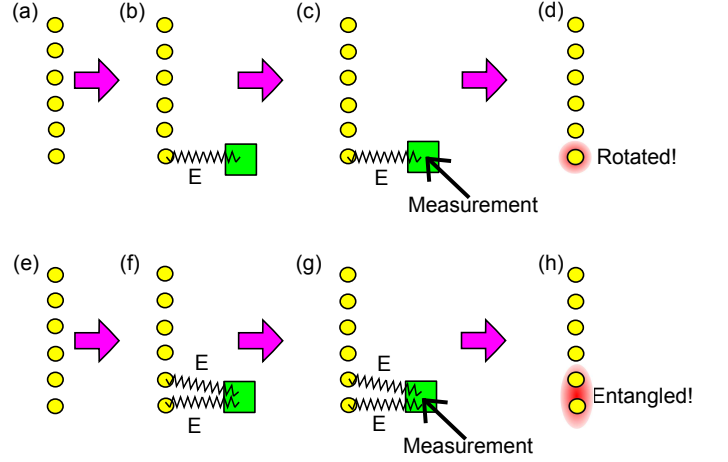


FIG. 1: (Color online.) The ancilla-driven quantum gates [17, 18]. Yellow circles are register qubits. Top line: a single-qubit rotation. (b) The ancilla (green square) is coupled to the qubit we want to rotate (say, the bottommost one) through the interaction E , which is represented by the black zigzag line. (c) After the interaction, the ancilla is projected onto a certain direction (represented by the solid black arrow). (d) The measurement backaction rotates the bottommost qubit of the register by the desired angle. Bottom line: a two-qubit entangling gate. (f) The ancilla (green square) is coupled to two qubits (say, the two bottommost ones) through the interaction E , which is the same as the interaction used in the top line. The interaction is represented by the black zigzag line. (g) After the interaction, the ancilla is projected (solid black arrow). (h) The measurement backaction causes the desired entangling gate between the two bottommost qubits of the register.

In Refs. [17, 18], it was shown that the interaction

$$CZ + SWAP \equiv |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| - |11\rangle\langle 11|$$

also enables the ancilla-driven universal quantum computation (see Fig. 3). Interestingly, interactions which enable the ancilla-driven universal quantum computation are, apart from local unitaries, only two types: CZ and CZ+SWAP [17, 18].

III. ENTANGLEMENT-FIDELITY RELATION FOR THE ONE-WAY MODEL

Before giving main results of this paper, let us also review the entanglement-fidelity relation [16] for the one-way quantum computation, because, as we will see in the next section, some of ancilla-driven gates have exactly the same entanglement-fidelity relation as that for the one-way model.

In Ref. [16], the entanglement-fidelity relation was studied for the elementary process (Fig. 4) of the one-way single-qubit rotation and the controlled-not gate, assuming that the projective measurement is inaccurate in the sense that the direction to which the qubit is projected

calculated as

$$F \equiv \mathbb{E} \left[\left| \langle \phi_{0,0} | \phi_{\epsilon,\delta} \rangle \right|^2 \right] = \cos^2 \frac{\epsilon}{2} + \text{Tr}^2(\hat{\rho}_b \hat{Z}_b) \sin^2 \frac{\epsilon}{2}, \quad (3)$$

where $\mathbb{E}[\cdot]$ means the average over all measurement histories,

$$\hat{\rho}_b \equiv \text{Tr}_b(|\psi\rangle\langle\psi|)$$

is the reduced density operator for the bottommost qubit, Tr_b is the trace over all register qubits except for the bottommost qubit, $|\psi\rangle$ is the initial state of the register (i.e., the state of qubits in the green ellipse of Fig. 4 (a)), and \hat{Z}_b is the Pauli z operator acting on the bottommost qubit.

Let us define the amount S ($0 \leq S \leq 1$) of entanglement

$$S \equiv 2[1 - \text{Tr}(\hat{\rho}_b^2)] \quad (4)$$

between the bottommost qubit and other register qubits (i.e., qubits in the blue ellipse of Fig. 4 (a)). This entanglement is indicated by the red bond in Fig. 4 (a). If the bottommost qubit and other register qubits are not entangled, $S = 0$, whereas if they are maximally entangled, $S = 1$.

By noticing that $1 - S$ is equal to the square length of the Bloch vector for $\hat{\rho}_b$:

$$1 - S = \text{Tr}^2(\hat{X}_b \hat{\rho}_b) + \text{Tr}^2(\hat{Y}_b \hat{\rho}_b) + \text{Tr}^2(\hat{Z}_b \hat{\rho}_b),$$

we obtain the obvious inequality

$$1 - S \geq \text{Tr}^2(\hat{Z}_b \hat{\rho}_b).$$

The intuitive meaning of this inequality is that if $\hat{\rho}_b$ is more mixed, i.e., the entanglement is larger, the length of the Bloch vector (and hence the z -component of the Bloch vector) becomes shorter. From this inequality, an upper bound of the right-hand-side of Eq. (3) is given as

$$\cos^2 \frac{\epsilon}{2} + \text{Tr}^2(\hat{\rho}_b \hat{Z}_b) \sin^2 \frac{\epsilon}{2} \leq \cos^2 \frac{\epsilon}{2} + (1 - S) \sin^2 \frac{\epsilon}{2},$$

and hence we finally obtain the desired inequality

$$F \equiv \mathbb{E} \left[\left| \langle \phi_{0,0} | \phi_{\epsilon,\delta} \rangle \right|^2 \right] \leq 1 - S \sin^2 \frac{\epsilon}{2}. \quad (5)$$

Note that this inequality gives the optimal upper bound for F , since the equality is always achieved for any given S ($0 \leq S \leq 1$) by taking

$$\hat{\rho}_b = \frac{\sqrt{1-S}}{2} \hat{Z}_b + \frac{1}{2} \hat{1}_b,$$

where $\hat{1}_b$ is the identity operator on the bottommost qubit.

The inequality (5) is the relation between entanglement among register qubits and the gate fidelity in the

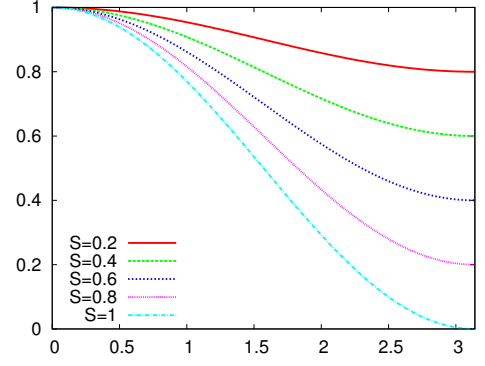


FIG. 5: (Color online.) $1 - S \sin^2 \frac{\epsilon}{2}$ as a function of ϵ for $S = 0.2, 0.4, 0.6, 0.8$, and 1 .

inaccurate one-way quantum computation. The right-hand-side of Eq. (5) is plotted as a function of ϵ for various S in Fig. 5. We can see that for a fixed ϵ , larger S makes F smaller. As is discussed in Ref. [16], S is often very large in many quantum algorithms [2, 21].

Although we have used the purity Eq. (4) as a measure of entanglement, we can derive a similar entanglement-fidelity relation by using the von Neumann entropy S_v ($0 \leq S_v \leq 1$)

$$S_v \equiv -\text{Tr}(\hat{\rho}_b \log_2 \hat{\rho}_b)$$

as a measure of entanglement. If $\hat{\rho}_b$ is maximally mixed, $S_v = 1$, whereas $S_v = 0$ if $\hat{\rho}_b$ is pure. By a straightforward calculation,

$$\begin{aligned} S_v &= -\frac{1+r}{2} \log_2 \frac{1+r}{2} - \frac{1-r}{2} \log_2 \frac{1-r}{2} \\ &\leq -\frac{1+|C_z|}{2} \log_2 \frac{1+|C_z|}{2} - \frac{1-|C_z|}{2} \log_2 \frac{1-|C_z|}{2} \\ &\equiv f(|C_z|), \end{aligned}$$

where r is the length

$$r \equiv \sqrt{\text{Tr}^2(\hat{X}_b \hat{\rho}_b) + \text{Tr}^2(\hat{Y}_b \hat{\rho}_b) + \text{Tr}^2(\hat{Z}_b \hat{\rho}_b)}$$

of the Bloch vector of $\hat{\rho}_b$ and

$$C_z \equiv \text{Tr}(\hat{\rho}_b \hat{Z}_b).$$

Let f^{-1} be the inverse of f . Then,

$$|C_z| \leq f^{-1}(S_v).$$

and therefore

$$|C_z|^2 \leq (f^{-1}(S_v))^2.$$

Inserting this inequality into Eq. (3), we obtain

$$F \leq 1 - [1 - (f^{-1}(S_v))^2] \sin^2 \frac{\epsilon}{2}.$$

The equality in this inequality is always achievable for any S_v , and therefore this is the optimal upper bound.

In Fig. 6, we plot $1 - (f^{-1}(S_v))^2$ as a function of S_v . We can see that $1 - (f^{-1}(S_v))^2$ is a monotonically increasing function of S_v . Therefore, we can also say that if the entanglement is strong in terms of S_v , the inaccurate measurements make the gate fidelity low.

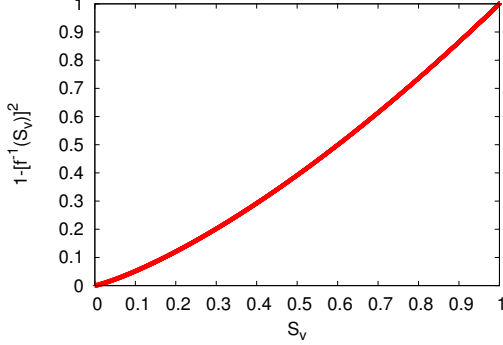


FIG. 6: (Color online.) $1 - [f^{-1}(S_v)]^2$ as a function of S_v .

IV. ENTANGLEMENT-FIDELITY RELATION FOR THE ANCILLA-DRIVEN MODEL

A. Single-qubit rotation

Let us explore a similar entanglement-fidelity relation for the ancilla-driven single-qubit rotation (Fig. 2 (a) and Fig. 3 (a)). As is mentioned before, we obtain the same inequality Eq. (5) for the ancilla-driven single-qubit rotation.

In order to see it, let us first consider the ancilla-driven single-qubit rotation with the CZ+SWAP interaction (Fig. 3 (a)). It is immediate to see that this circuit is equivalent to that in Fig. 4 (e). Therefore, we obtain the same inequality, Eq. (5).

Second, let us consider the ancilla-driven single-qubit rotation with the CZ interaction (Fig. 2 (a)). Again, we can show that this circuit is equivalent to that in Fig. 4 (e), since the application of $\hat{H}_R \hat{H}_A$ on states $|0\rangle_R |+\rangle_A$ or $|1\rangle_R |-\rangle_A$ is equivalent to that of SWAP operation on the same states:

$$\begin{aligned} \hat{H}_R \hat{H}_A |0\rangle_R |+\rangle_A &= |+\rangle_R |0\rangle_A = \hat{S} \hat{W} \hat{A} P_{R,A} |0\rangle_R |+\rangle_A, \\ \hat{H}_R \hat{H}_A |1\rangle_R |-\rangle_A &= |-\rangle_R |1\rangle_A = \hat{S} \hat{W} \hat{A} P_{R,A} |1\rangle_R |-\rangle_A. \end{aligned}$$

In short, Eq. (5) also gives the entanglement-fidelity relation for the ancilla-driven single-qubit rotation. In this case, S is the amount of entanglement between the qubit we want to rotate and other register qubits in the initial state.

B. Two-qubit entangling gate with the CZ interaction

Let us next consider the ancilla-driven two-qubit entangling gate. We first consider the CZ interaction (Fig. 2 (b)). Let the input state be

$$|\psi\rangle_r |+\rangle_a = \left[\sum_{z_1=0}^1 \sum_{z_2=0}^1 \eta_{z_2, z_1} |\eta_{z_2, z_1}\rangle_o |z_2\rangle_2 |z_1\rangle_1 \right] |+\rangle_a, \quad (6)$$

where $|\psi\rangle_r$ is the register state, $|+\rangle_a$ is the ancilla state, $\eta_{z_1, z_2} \in \mathbb{C}$, $|z_i\rangle_i$ is the state of i th qubit in the register, and $|\eta_{z_2, z_1}\rangle_o$ is the state of register qubits other than first and second qubits. By a straightforward calculation, the state immediately before the measurement of the ancilla is

$$\begin{aligned} &\eta_{00} |\eta_{00}\rangle_o |+\rangle_2 |+\rangle_1 |+\rangle_a + \eta_{01} |\eta_{01}\rangle_o |+\rangle_2 |-\rangle_1 |-\rangle_a \\ &+ \eta_{10} |\eta_{10}\rangle_o |-\rangle_2 |+\rangle_1 |+\rangle_a - \eta_{11} |\eta_{11}\rangle_o |-\rangle_2 |-\rangle_1 |-\rangle_a. \end{aligned}$$

If the measurement is accurate, we obtain the desired output

$$|\phi_{0,0}^j\rangle \equiv \hat{X}_1^j \hat{H}_1 \hat{H}_2 \hat{C} \hat{Z}_{1,2} |\psi\rangle_r$$

with the correction \hat{X}_1^j which depends on the measurement result $j = 0, 1$. However, if the measurement is inaccurate in the sense that the ancilla is projected onto

$$\begin{aligned} |\tilde{0}\rangle &\equiv \cos \frac{\epsilon}{2} |0\rangle + \sin \frac{\epsilon}{2} e^{-i\delta} |1\rangle \\ |\tilde{1}\rangle &\equiv \sin \frac{\epsilon}{2} |0\rangle - \cos \frac{\epsilon}{2} e^{-i\delta} |1\rangle, \end{aligned}$$

we can show by a straightforward calculation that the output state is

$$|\phi_{\epsilon,\delta}^j\rangle \equiv \left(\cos \frac{\epsilon}{2} + (-1)^j \hat{X}_1 e^{(-1)^j i\delta} \sin \frac{\epsilon}{2} \right) |\phi_{0,0}^j\rangle_r.$$

Note that we obtain the same error operation as that in Eq. (2). Therefore, by a similar calculation as that for the derivation of Eq. (5), we obtain Eq. (5). In summary, Eq. (5) is also the entanglement-fidelity relation for the ancilla-driven two-qubit entangling gate with the CZ interaction.

C. Two-qubit entangling gate with the CZ+SWAP interaction

Finally, let us consider the CZ+SWAP interaction (Fig. 3 (b)). In this case, the entanglement-fidelity relation we will obtain is Eq. (8), which is different from Eq. (5) in the sense that the two-body entanglement appears. However we can still say from Eq. (8) that the strong entanglement causes low gate fidelity.

We assume the same input state Eq. (6). The state immediately before the final measurement is straightforwardly calculated as

$$\begin{aligned} &\eta_{00} |\eta_{00}\rangle_o |0\rangle_2 |0\rangle_1 |+\rangle_a + \eta_{01} |\eta_{01}\rangle_o |1\rangle_2 |0\rangle_1 |-\rangle_a \\ &+ \eta_{10} |\eta_{10}\rangle_o |0\rangle_2 |1\rangle_1 |-\rangle_a - \eta_{11} |\eta_{11}\rangle_o |1\rangle_2 |1\rangle_1 |+\rangle_a. \end{aligned}$$

If the measurement is accurate, we obtain

$$|\phi_{0,0}^j\rangle \equiv (\hat{Z}_1\hat{Z}_2)^j SWAP_{1,2} \hat{C}Z_{1,2}|\psi\rangle.$$

If the measurement is inaccurate, we obtain

$$|\phi_{\epsilon,\delta}^j\rangle \equiv \left(\cos\frac{\epsilon}{2} + (-1)^j \hat{Z}_1\hat{Z}_2 \sin\frac{\epsilon}{2} e^{(-1)^j i\delta}\right) |\phi_{0,0}^j\rangle.$$

Note that in this case, the error is, intuitively, the application of $\hat{Z}_1\hat{Z}_2$ with the weight $\sin\frac{\epsilon}{2}$. This is slightly different from Eq. (2). By a similar calculation as that for deriving Eq. (5), we obtain

$$F = \cos^2\frac{\epsilon}{2} + \text{Tr}^2(\hat{Z}_1\hat{Z}_2\hat{\rho}_{1,2}) \sin^2\frac{\epsilon}{2},$$

where $\hat{\rho}_{1,2}$ is the reduced density operator for first and second qubits of the input state of the register.

As a measure of entanglement, let us adopt the von Neumann entropy S_{v2} ($0 \leq S_{v2} \leq 2$)

$$S_{v2}(\hat{\rho}) = -\text{Tr}(\hat{\rho} \log_2 \hat{\rho}),$$

where $\hat{\rho}$ is a two-qubit state. The subscript 2 of S_{v2} indicates two-qubit entanglement. If a two-qubit state $\hat{\rho}$ is maximally entangled with other qubits, $S_{v2}(\hat{\rho}) = 2$, whereas if it is separable from other qubits, $S_{v2}(\hat{\rho}) = 0$.

As is shown in Appendix,

$$\begin{aligned} S_{v2}(\hat{\rho}) &\leq -\frac{1+|C_{zz}|}{2} \log_2 \left(\frac{1+|C_{zz}|}{4} \right) \\ &\quad -\frac{1-|C_{zz}|}{2} \log_2 \left(\frac{1-|C_{zz}|}{4} \right) \\ &\equiv g(|C_{zz}|) \end{aligned} \quad (7)$$

for any $\hat{\rho}$, where

$$C_{zz} \equiv \text{Tr}(\hat{\rho} \hat{Z} \otimes \hat{Z}).$$

Let us denote the inverse of g by g^{-1} . Then, Eq. (7) gives

$$|C_{zz}| \leq g^{-1}(S_{v2})$$

for $1 \leq S_{v2} \leq 2$ (note that g^{-1} is not defined for $0 \leq S_{v2} \leq 1$). Thus we obtain

$$F \leq 1 - [1 - (g^{-1}(S_{v2}))^2] \sin^2\frac{\epsilon}{2} \quad (8)$$

for $1 \leq S_{v2} \leq 2$. The equality in this inequality is always achievable for any $1 \leq S_{v2} \leq 2$, and therefore this is the optimal upper bound.

In Fig. 7, we plot $1 - (g^{-1}(S_{v2}))^2$ as a function of S_{v2} . We can see that $1 - (g^{-1}(S_{v2}))^2$ is a monotonically increasing function of S_{v2} . Therefore we can say that if the entanglement is strong ($S_{v2} \geq 1$), the inaccurate measurements make the gate fidelity low.

Let us also investigate the case $S_{v2} < 1$. In this case, we cannot say anything about F , since any function of S_{v2} does not provide any nontrivial upper bound for $|C_{zz}|^2$ if $S_{v2} < 1$. Indeed, let us consider the state

$$\hat{\rho}_\lambda = \lambda|00\rangle\langle 00| + (1-\lambda)|11\rangle\langle 11|,$$

where $0 \leq \lambda \leq 1$. Although $S_{v2}(\hat{\rho}_\lambda)$ can take any value between 0 and 1 by changing λ , $|C_{zz}|$ is always 1 for any λ . Therefore, for $S_{v2} < 1$, there is no meaningful relation between S_{v2} and F .

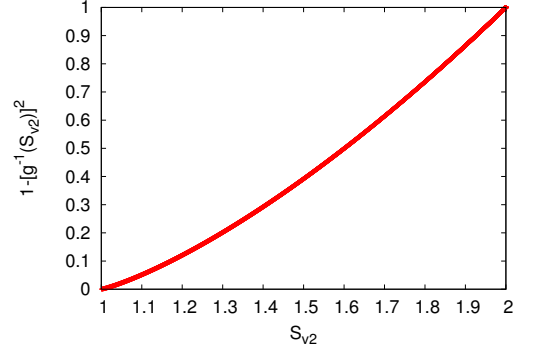


FIG. 7: (Color online.) $1 - [g^{-1}(S_{v2})]^2$ as a function of S_{v2} .

V. SUMMARY AND DISCUSSION

In this paper, we have studied how the entanglement among register qubits affects the gate fidelity of the inaccurate ancilla-driven quantum computation. By generalizing the previous result about the entanglement-fidelity relation in the inaccurate one-way quantum computation [16], we have shown similar entanglement-fidelity relations for the inaccurate ancilla-driven quantum computation. Our results are (I) For the ancilla-driven single-qubit rotation, and for the ancilla-driven two-qubit entangling gate with the CZ interaction, we obtain the entanglement-fidelity relation as given in Eq. (1). (II) For the ancilla-driven two-qubit entangling gate with the CZ+SWAP interaction, we obtain the entanglement-fidelity relation Eq. (8) which is slightly different from Eq. (1). These relations imply that if the entanglement is strong, the inaccurate measurements make the gate fidelity low in the ancilla-driven quantum computation.

As is mentioned in Ref. [16], the error model considered here is a kind of error that can ultimately be recovered with the quantum error-correcting code. Therefore, our entanglement-fidelity relations are of use for studying the stability of a bare quantum computation in order to obtain valuable feedbacks for the study of general fault-tolerant schemes, to develop the made-to-measure error-correcting codes, to estimate the threshold value of the error-correction, and to help experimentalists who want to perform proof-of-principle experiments with few qubits.

It is very important to study the difference between the one-way model and the ancilla-driven model from the view point of our entanglement-fidelity relations. Although these two models share many similarities, one interesting difference that has been revealed in this paper is that not only the single-qubit entanglement but also the two-qubit entanglement are related to the gate fidelity in the ancilla-driven case. Therefore, in addition to the physical constraints come from the specific experimental setups in the laboratory, the amount of the two-qubit entanglement in the register can also be one criterion for

choosing the one-way model or the ancilla-driven model. Detailed studies according to this direction would be a subject of the future study.

Appendix: Proof of Eq. (7)

Let

$$H(\hat{\rho}||\hat{\sigma}) \equiv \text{Tr}(\hat{\rho} \log_2 \hat{\rho}) - \text{Tr}(\hat{\rho} \log_2 \hat{\sigma})$$

be the relative entropy between $\hat{\rho}$ and $\hat{\sigma}$. As is well known [1], a Completely-Positive and Trace-Preserving (CPTP) map \mathcal{E} cannot increase the amount of the relative entropy:

$$H(\mathcal{E}(\hat{\rho})||\mathcal{E}(\hat{\sigma})) \leq H(\hat{\rho}||\hat{\sigma}). \quad (\text{A.1})$$

With the notation $\rho_{ab} = \langle ab|\hat{\rho}|ab\rangle$ for diagonal elements, let us consider the CPTP map

$$\begin{aligned} \mathcal{E}(\hat{\rho}) = & \rho_{00}|00\rangle\langle 00| + \rho_{01}|01\rangle\langle 01| \\ & + \rho_{10}|10\rangle\langle 10| + \rho_{11}|11\rangle\langle 11|. \end{aligned}$$

If we take

$$\hat{\sigma} = \frac{1}{4}\hat{1} \otimes \hat{1},$$

where $\hat{1}$ is the single-qubit identity operator, and replace within Eq. (A.1), we obtain:

$$\begin{aligned} \text{Tr}(\hat{\rho} \log_2 \hat{\rho}) \geq & \rho_{00} \log_2(\rho_{00}) + \rho_{11} \log_2(\rho_{11}) \\ & + \rho_{01} \log_2(\rho_{01}) + \rho_{10} \log_2(\rho_{10}). \end{aligned} \quad (\text{A.2})$$

Now since

$$\begin{aligned} C_{zz} &= 2(\rho_{00} + \rho_{11}) - 1 \\ &= 1 - 2(\rho_{01} + \rho_{10}), \end{aligned}$$

and $(x \mapsto x \log_2 x)$ is a convex function, maximum in the right-hand side of Eq. (A.2) for fixed C_{zz} is achieved for

$$\begin{aligned} \rho_{00} = \rho_{11} &= \frac{1 + |C_{zz}|}{4}, \\ \rho_{01} = \rho_{10} &= \frac{1 - |C_{zz}|}{4}. \end{aligned}$$

Substituting in Eq. (A.2) yields Eq. (7).

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